

Since the constant α is just a multiplicative scaling factor, one expects that, for various values of η , a linear correlation be found between $(\xi - \lambda)$ from Rathjen and Jiji and the present $\phi_1(\eta)$ with $\alpha = 1$. The constant of proportionality effectively determines the actual value of α . Now although equations (28)–(32) appear somewhat complex, the calculation is in fact very easy. The integrals of the complementary error function, the Hermite polynomials, and the B_n themselves, can all be calculated iteratively in n , and the series evaluated term by term. For $\beta = 0.1$, it was found that the difference between taking 10 and 20 terms was in the fourth significant figure, so 20 terms were considered sufficient for a rough comparison. For η in the range 2.25(0.05)2.8, a correlation coefficient of 0.999 was found, and the regression gave $\alpha = 0.18(5)$, with a very small intercept at about 2×10^{-5} , which should in theory be zero. Note that taking too large a value of η for comparison gives insufficient difference from the Neumann solution to rely on with any accuracy, and too small a value will be affected by higher order terms in the asymptotic expansion. Given this, the agreement is clearly very satisfactory, and bears out the form of the solution. The results are presented graphically in Fig. 2.

5. CONCLUSIONS

The leading term in the asymptotic approach of a two-dimensional Stefan problem to the Neumann solution has been found, in a form amenable to calculation. Agreement with available numerical results for the case of a right-angled corner is good.

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APPENDIX

The following results are needed in the foregoing analysis. The proofs are straightforward, and so are omitted. If

$$J_n = \int_{-\infty}^{\infty} \eta e^{-\eta^2} H_n(\eta) d\eta \quad (A1)$$

then

$$J_1 = \sqrt{\pi}, \quad \text{and} \quad J_n = 0, \quad \text{for } n = 0, 2, 3, 4, \dots \quad (A2)$$

If

$$I_n = \int_{-\infty}^{\infty} [1 - (\text{erf } \eta)^2] H_n(\eta) d\eta \quad (A3)$$

then

$$I_n = 0 \quad \text{if } n \text{ is odd}$$

and

$$I_n = \frac{(-1)^{n/2} n!}{(n+1)(n/2)! 2^{(n-3)/2} \sqrt{\pi}} \quad \text{if } n \text{ is even.} \quad (A4)$$

Heat transfer in the flow of a second-order fluid between two enclosed rotating discs

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NOMENCLATURE

$a_{i,j}$	covariant derivative of the covariant acceleration vector a_i
c_v	specific heat
d_{ij}	strain rate tensor
d_i^m	mixed strain rate tensor
E	Eckert number, $\Omega z_0^2 / c_v (T_b - T_a)$
G	dimensionless function of ζ
H	dimensionless function of ζ
k	thermal conductivity
L	dimensionless function of ζ
M	dimensionless function of ζ
Nu_a	average Nusselt number on the lower disc
Nu_b	average Nusselt number on the upper disc
Pr	Prandtl number, $\mu_1 c_v / k$
q_a	heat flux from the lower disc
q_b	heat flux from the upper disc
Q_a	amount of heat transfer from the lower disc
Q_b	amount of heat transfer from the upper disc
r	radial coordinate
Re_l	Reynolds number based on circulatory flow, $l / 2\pi \rho z_0 v_1$

Re_m	Reynolds number based on net radial outflow, $m / 2\pi \rho z_0 v_1$
Re_z	Reynolds number based on the gap length z_0 , $\Omega z_0^2 / v_1$
T	temperature
T^*	dimensionless temperature, $T c_v / v_1 \Omega$
T_a	temperature at the lower disc
T_a^*	dimensionless temperature at the lower disc
T_b	temperature at the upper disc
T_b^*	dimensionless temperature at the upper disc
u	radial velocity
U	dimensionless radial velocity, $u / \Omega z_0$
v	azimuthal velocity
$v_{i,j}$	covariant derivative of the covariant velocity vector v_i
v_i^m	covariant derivative of the contravariant velocity vector v^m
V	dimensionless azimuthal velocity, $v / \Omega z_0$
w	axial velocity
W	dimensionless axial velocity, $w / \Omega z_0$
z	axial coordinate
z_0	gap length between the lower and upper discs.

Greek symbols

ζ	dimensionless axial variable, z/z_0
θ	azimuthal coordinate
μ_1	Newtonian viscosity
μ_2	elastico-viscosity
μ_3	cross-viscosity
ν_1	kinematic Newtonian viscosity
ν_2	kinematic elastico-viscosity
ν_3	kinematic cross-viscosity
ρ	density of the fluid
τ_1	dimensionless parameter representing the elastico-viscous effect, ν_2/z_0^2
τ_2	dimensionless parameter representing the cross-viscous effect, ν_3/z_0^2

τ	sum of the parameters τ_1 and τ_2
τ_{ij}	stress tensor
τ_j^i	mixed stress tensor
ϕ	dimensionless function of ζ
Φ	viscous dissipation function
Φ^*	dimensionless viscous dissipation function
χ	dimensionless radial variable, r/z_0
χ_0	fixed dimensionless radius, r_0/z_0
χ_1	minimum value of χ for no-recirculation in the case of $Re_m > 0$
χ_2	maximum value of χ for no-recirculation in the case of $Re_m < 0$
ψ	dimensionless function of ζ
Ω	angular velocity.

INTRODUCTION

THE PROBLEM of the fluid flow between two finite enclosed rotating discs (enclosed in a cylindrical casing) or shrouded discs has important engineering applications as its generalization could be helpful in the study of heat transfer analysis of air cooling of turbine discs [1, 2] and the determination of the oil film temperature of pedestal bearings with centre feeding of lubricant [3]. Soo *et al.* [4] investigated the nature of heat transfer from an enclosed rotating disc. Sharma and Agarwal [5] reconsidered this problem using an improved formulation suggested by Sharma [6]. The flow of a non-Newtonian second-order fluid between two enclosed rotating discs has recently been considered by Sharma and Gupta [7].

The present note is an attempt to study the heat transfer in the problem considered by Sharma and Gupta [7] in the following two cases:

- (I) when the discs rotate in the same sense;
- (II) when the discs rotate in the opposite sense.

FORMULATION OF THE PROBLEM

The constitutive equation of an incompressible second-order fluid as suggested by Coleman and Noll [8] can be written as

$$\tau_{ij} = 2\mu_1 d_{ij} + 2\mu_2 e_{ij} + 4\mu_3 d_i^m d_{mj} \quad (1)$$

where

$$d_{ij} = \frac{1}{2}(v_{i,j} + v_{j,i}) \quad (2)$$

and

$$e_{ij} = \frac{1}{2}(a_{i,j} + a_{j,i}) + v_i^m v_{m,j}. \quad (3)$$

In a three-dimensional cylindrical set of coordinates (r, θ, z) the system consists of two finite rotating discs of radius r_s (coinciding with planes $z = 0$ and z_0) rotating with constant angular velocities $s_1\Omega$ and $s_2\Omega$, respectively (where s_1 and s_2 are constant parameters), in an incompressible second-order fluid forming part of a co-axial cylindrical casing. The symmetrical radial steady outflow has a small mass rate ' m ' of radial outflow (' $-m$ ' for radial inflow). The inlet condition is taken as a simple radial source flow along the z -axis starting from radius r_0 . The lower disc $z = 0$ is maintained at a constant temperature T_a while the upper disc $z = z_0$ is maintained at T_b .

The energy equation describing the transport of thermal energy is

$$\rho c_v \frac{DT}{Dt} = k \nabla^2 T + \Phi \quad (4)$$

where

$$\Phi = \tau_j^i d_i^j. \quad (5)$$

The boundary conditions on the velocity profile and temperature are

$$\left. \begin{aligned} u &= 0, & v &= rs_1\Omega, & w &= 0, & T &= T_a & \text{at } z &= 0 \\ u &= 0, & v &= rs_2\Omega, & w &= 0, & T &= T_b & \text{at } z &= z_0. \end{aligned} \right\} \quad (6)$$

In non-dimensional form the velocity field is taken as [5]

$$\left. \begin{aligned} U &= -\chi H'(\zeta) + \frac{Re_m}{Re_z} \frac{M'(\zeta)}{\chi} \\ V &= \chi G(\zeta) + \frac{Re_1}{Re_z} \frac{L(\zeta)}{\chi} \\ W &= 2H(\zeta). \end{aligned} \right\} \quad (7)$$

The expressions for H , G , L and M' as found in ref. [7] are

$$H = \frac{Re_z}{60} [s_1^2(\zeta^5 - 5\zeta^4 + 7\zeta^3) + s_2^2(\zeta^5 - 3\zeta^3 + 2\zeta^2) - s_1 s_2(2\zeta^5 - 5\zeta^4 + 4\zeta^3 - \zeta^2)] \quad (8a)$$

$$\begin{aligned}
G = & [s_1 + (s_2 - s_1)\zeta] + \frac{Re_z^2}{6300} [s_1^3(20\zeta^7 - 140\zeta^6 + 357\zeta^5 - 420\zeta^4 + 210\zeta^3 - 27\zeta) \\
& - s_2^3(20\zeta^7 - 63\zeta^5 + 35\zeta^4 + 8\zeta) - s_1^2s_2(60\zeta^7 - 280\zeta^6 + 441\zeta^5 - 280\zeta^4 + 70\zeta^3 - 11\zeta) \\
& + s_2^2s_1(60\zeta^7 - 140\zeta^6 + 21\zeta^5 + 175\zeta^4 - 140\zeta^3 + 24\zeta)] + \tau \frac{Re_z^2}{30} [s_2^3(\zeta^5 - 3\zeta^3 + 2\zeta^2) \\
& - s_1^3(\zeta^5 - 5\zeta^4 + 7\zeta^3 - 3\zeta^2) + s_1^2s_2(3\zeta^5 - 10\zeta^4 + 11\zeta^3 - 4\zeta^2) - s_2^2s_1(3\zeta^5 - 5\zeta^4 + \zeta^3 + \zeta^2)] \quad (8b)
\end{aligned}$$

$$L = Re_z \left(\frac{Re_m}{Re_l} \right) \left[\frac{1}{3} \{s_1(3\zeta^5 - 10\zeta^4 + 10\zeta^3 - 3\zeta) - s_2(3\zeta^5 - 5\zeta^4 + 2\zeta)\} + 4\tau(s_2 - s_1)(2\zeta^3 - 3\zeta^2 + \zeta) \right] \quad (8c)$$

$$\begin{aligned}
M' = & 6(\zeta - \zeta^2) + \frac{Re_z^2}{4200} [s_1^2(60\zeta^8 - 300\zeta^7 + 588\zeta^6 - 294\zeta^5 - 630\zeta^4 + 840\zeta^3 - 263\zeta^2 - \zeta) \\
& + s_2^2(60\zeta^8 - 180\zeta^7 + 168\zeta^6 - 294\zeta^5 + 420\zeta^4 - 263\zeta^2 + 89\zeta) \\
& - 2s_1s_2(60\zeta^8 - 240\zeta^7 + 238\zeta^6 + 126\zeta^5 - 105\zeta^4 - 280\zeta^3 + 247\zeta^2 - 46\zeta)] \\
& - \tau \frac{Re_z^2}{525} [s_1^2(105\zeta^6 - 595\zeta^4 + 1190\zeta^3 - 939\zeta^2 + 239\zeta) + s_2^2(105\zeta^6 - 630\zeta^5 + 980\zeta^4 - 910\zeta^3 + 636\zeta^2 - 181\zeta) \\
& - s_1s_2(210\zeta^6 - 630\zeta^5 + 385\zeta^4 + 280\zeta^3 - 303\zeta^2 + 58\zeta)] + \frac{8}{3}\tau^2 Re_z^2(s_2 - s_1)^2(5\zeta^4 - 10\zeta^3 + 6\zeta^2 - \zeta). \quad (8d)
\end{aligned}$$

The energy equation, equation (4), takes the form

$$\left(U \frac{\partial T^*}{\partial \chi} + W \frac{\partial T^*}{\partial \zeta} \right) = \frac{1}{Pr Re_z} \left(\frac{\partial^2 T^*}{\partial \chi^2} + \frac{1}{\chi} \frac{\partial T^*}{\partial \chi} + \frac{\partial^2 T^*}{\partial \zeta^2} \right) + \Phi^*. \quad (9)$$

SOLUTION OF THE PROBLEM

Using velocity field (7) in equation (9) and neglecting terms of order $(Re_m/Re_z)^2$ and higher, we get

$$\begin{aligned}
& \left[\left(-\chi H' + \frac{1}{\chi} \frac{Re_m}{Re_z} M' \right) \frac{\partial T^*}{\partial \chi} + 2H \frac{\partial T^*}{\partial \zeta} \right] \\
& = \frac{1}{Pr Re_z} \left[\frac{\partial^2 T^*}{\partial \chi^2} + \frac{1}{\chi} \frac{\partial T^*}{\partial \chi} + \frac{\partial^2 T^*}{\partial \zeta^2} \right] + \frac{1}{2} \left[24H' + \frac{4Re_m}{Re_z} \left(\frac{Re_l}{Re_m} G'L - H''M' \right) \right. \\
& \quad + 48\tau_1 Re_z(H'^3 + HH'H'') + 2Re_m \tau_1 \left\{ \frac{Re_l}{Re_m} (6H'L'G' + 6LG'H'' + 2HG'L' + 2HLE'G'') \right. \\
& \quad \left. \left. + (-6H'H''M'' - 2HH''M'' - 2M'H''^2 - 2HM''H'' + 4M'G'^2) \right\} + 48\tau_2 Re_z H'^3 \right. \\
& \quad \left. + 2Re_m \tau_2 \left\{ (-3M'H''^2 - 6H'H''M'' + 3M'G'^2) + \frac{Re_l}{Re_m} (6H'L'G' + 6LG'H'') \right\} \right] \\
& \quad + \frac{1}{2} \chi^2 [(2H''^2 + G'^2) + \tau_1 Re_z (4H'H''^2 + 4H'G'^2 + 4HH''H'' + 4HG'G'') + 6\tau_2 Re_z (H'H''^2 + H'G'^2)] \quad (10)
\end{aligned}$$

where primes denote differentiation with respect to ζ . The appropriate form for T^* as suggested from equation (10) is

$$T^* = T_a^* + \phi(\zeta) + \chi^2 \psi(\zeta). \quad (11)$$

Following Sharma and Agarwal [5] and using the values of H , G , L and M' , the dimensionless form of the temperature is obtained as

$$\begin{aligned}
\frac{T - T_a}{T_b - T_a} & = \frac{T^* - T_a^*}{T_b^* - T_a^*} \\
& = \zeta + \frac{Pr Re_z^2}{12600} [s_1^2(10\zeta^7 - 70\zeta^6 + 147\zeta^5 - 105\zeta^4 + 18\zeta) + s_2^2(10\zeta^7 - 63\zeta^5 + 74\zeta^4 - 17\zeta) \\
& \quad - s_1s_2(20\zeta^7 - 70\zeta^6 + 84\zeta^5 - 35\zeta^4 + \zeta)] + E Pr \left[\frac{1}{6} (s_2 - s_1)^2 (\zeta^4 - 2\zeta^3 + \zeta) \right. \\
& \quad + \frac{Re_m}{30} \{ 3Pr(s_2 - s_1)^2 (2\zeta^6 - 6\zeta^5 + 5\zeta^4 - \zeta) - 2\{s_1^2(\zeta^6 - 9\zeta^5 + 21\zeta^4 - 27\zeta^3 + 18\zeta^2 - 4\zeta) \\
& \quad + s_2^2(\zeta^6 + 3\zeta^5 - 9\zeta^4 + 13\zeta^3 - 12\zeta^2 + 4\zeta) + 2s_1s_2(-\zeta^6 + 3\zeta^5 - 6\zeta^4 + 7\zeta^3 - 3\zeta^2)\} \\
& \quad \left. - \{60\tau_1(\zeta^4 - 2\zeta^3 + 2\zeta^2 - \zeta) + 15\tau_2(5\zeta^4 - 10\zeta^3 + 8\zeta^2 - 3\zeta)\}(s_2 - s_1)^2 \} + \frac{1}{2} \chi^2 (s_2 - s_1)^2 \zeta(1 - \zeta) \right]. \quad (12)
\end{aligned}$$

The average Nusselt numbers on the lower and upper discs are

$$Nu_a = \frac{Q_a z_0}{(T_b - T_a)k}$$

$$= - \left[1 + \frac{Pr Re_z^2}{12600} (18s_1^2 - 17s_2^2 - s_1 s_2) + E Pr \left\{ \frac{1}{6} (s_2 - s_1)^2 + \frac{Re_m}{30} \{ -3P(s_2 - s_1)^2 + 8s_1^2 - 8s_2^2 + 60\tau_1(s_2 - s_1)^2 + 45\tau_2(s_2 - s_1)^2 \} + \frac{1}{4} (\chi^2 + \chi_0^2)(s_2 - s_1)^2 \right\} \right] \quad (13)$$

and

$$Nu_b = \frac{Q_b z_0}{(T_b - T_a)k}$$

$$= - \left[1 + \frac{Pr Re_z^2}{12600} (-17s_1^2 + 18s_2^2 - s_1 s_2) + E Pr \left\{ -\frac{1}{6} (s_2 - s_1)^2 + \frac{Re_m}{30} \{ 3P(s_2 - s_1)^2 + 8s_1^2 - 8s_2^2 - 60\tau_1(s_2 - s_1)^2 - 45\tau_2(s_2 - s_1)^2 \} - \frac{1}{4} (\chi^2 + \chi_0^2)(s_2 - s_1)^2 \right\} \right] \quad (14)$$

where

$$Q_a = \frac{1}{\pi(\chi^2 - \chi_0^2)} \int_{\chi_0}^{\chi} 2\pi\chi q_a d\chi \quad (15)$$

and

$$Q_b = \frac{1}{\pi(\chi^2 - \chi_0^2)} \int_{\chi_0}^{\chi} 2\pi\chi q_b d\chi \quad (16)$$

are the amounts of heat transfer; $q_a \{ = -(k/z_0)(\partial T/\partial \zeta)_{\zeta=0} \}$ and $q_b \{ = -(k/z_0)(\partial T/\partial \zeta)_{\zeta=1} \}$ the heat fluxes from the lower and upper discs, respectively.

RESULTS AND DISCUSSIONS

The dimensionless form of radii at which there is no-recirculation for the cases of net radial outflow ($Re_m > 0$) and net radial inflow ($Re_m < 0$), respectively, satisfy the following conditions

$$\text{I} \quad Re_m > 0; \quad \left(\frac{\partial U}{\partial \zeta} \right)_{\zeta=0} \geq 0, \quad \left(\frac{\partial U}{\partial \zeta} \right)_{\zeta=1} \leq 0, \quad (17)$$

$$\text{II} \quad Re_m (= -Re_n) < 0; \quad \left(\frac{\partial U}{\partial \zeta} \right)_{\zeta=0} \leq 0, \quad \left(\frac{\partial U}{\partial \zeta} \right)_{\zeta=1} \geq 0. \quad (18)$$

In case I ($s_1 = s_2 = 1$), equations (17) and (18) show that there is no-recirculation for any τ in both the cases of net radial outflow and inflow as it is almost a solid rotation. In case II ($s_1 = 1, s_2 = -1$), the minimum and maximum values of χ for no-

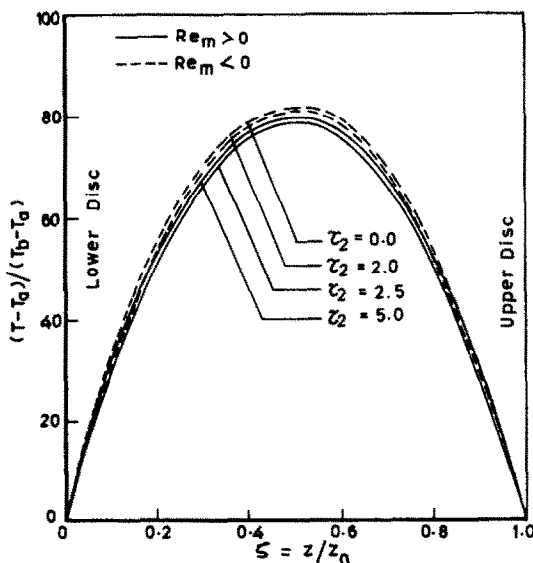


FIG. 1. Variation of temperature with τ_2 at $\chi = 20$.

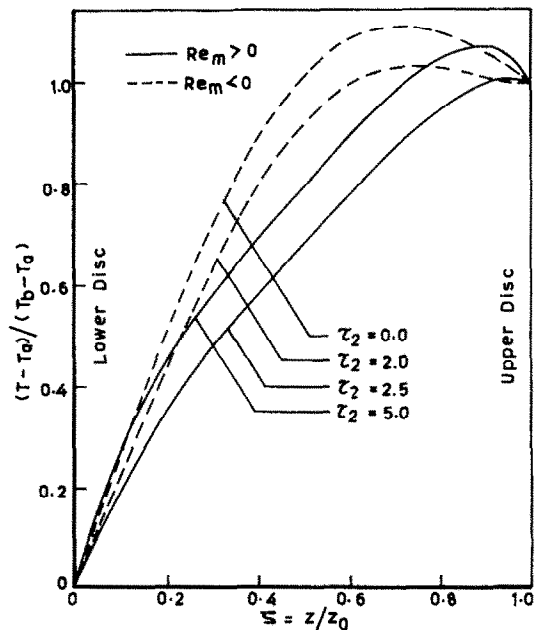
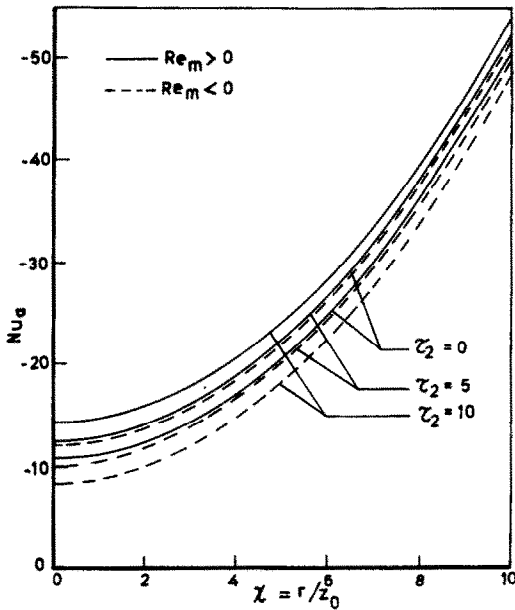
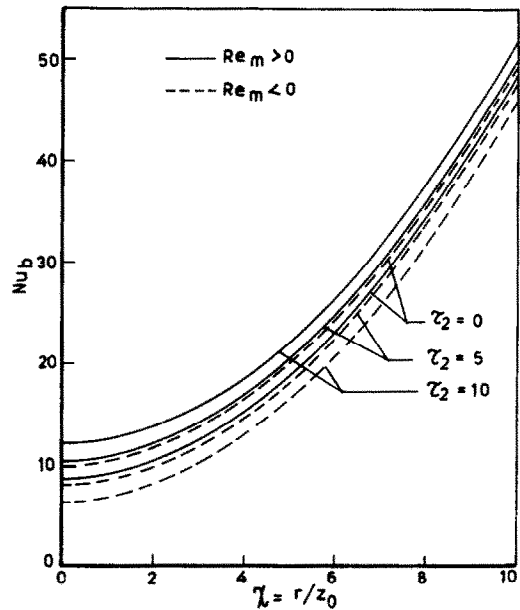


FIG. 2. Variation of temperature with τ_2 at $\chi = 1$.

FIG. 3. Variation of Nusselt number with τ_2 on the lower disc.FIG. 4. Variation of Nusselt number with τ_2 on the upper disc.

recirculation, namely, χ_1 and χ_2 obtained from equations (17) and (18), respectively, are given by

$$\frac{\chi_1^2}{Re_m} = \frac{Re_z^2(6720\tau^2 + 232\tau + 1) - 6300}{70 Re_z^2} \quad (19)$$

$$\frac{\chi_2^2}{Re_n} = \frac{6300 - Re_z^2(6720\tau^2 + 232\tau + 1)}{70 Re_z^2} \quad (20)$$

Hence in case II at $Re_z = 0.5$ the necessary conditions to be satisfied by the flows that admits certain radii at which there is no-recirculation, are respectively given by $\tau > 1.91$ for net radial outflow and $\tau < 1.91$ for net radial inflow.

For case I ($s_1 = s_2 = 1$), the temperature variation with ζ is linear as is evident from equation (12) and the Nusselt numbers are constants, equal to 1 and -1 throughout the entire radial region on the lower and upper discs, respectively.

For case II ($s_1 = 1, s_2 = -1$), Figs. 1 and 2 depict the behaviour of the temperature profile with τ_2 (based on the relation $\tau_1 = \alpha\tau_2$, where $\alpha = -0.2$ as for a 5.46% poly-iso-butylene type solution in cetane at 30°C [9]) for $Pr = 20, E = 0.02, Re_z = 0.5$ at $\chi = 20$ and 1 in the regions of recirculation (no-recirculation for $Re_m > 0$) and no-recirculation (recirculation for $Re_m > 0$), respectively. Figure 1 shows that the temperature curve is elliptic being maximum approximately in the middle of the gap length while the shape is different in Fig. 2 giving the maxima near the upper disc. It is found that an increase in τ_2 increases the temperature in both the regions throughout the gap length for a net radial outflow while in case of net radial inflow the behaviour is just opposite and the temperature is less than that in the case of a Newtonian fluid.

Variation of Nusselt number Nu_a and Nu_b with χ in the case of $s_1 = 1, s_2 = -1$ for $Pr = 20, E = 0.02, Re_z = 0.5$ and $\chi_0 = 5$ has been represented through Figs. 3 and 4, respectively. It is evident that the average Nusselt number on the lower disc is negative throughout the entire radial region and increases in magnitude with an increase in τ_2 for net radial outflow while it decreases in magnitude for net radial inflow (Fig. 3). The behaviour of the average Nusselt number Nu_b (which is positive) is similar to Nu_a . Since Nu_a is negative, the fluid is giving heat to the lower disc (at constant temperature T_c) due to dissipation [4]. Figures 3 and 4 also show that the heat transfer from the lower and upper discs in the case of a Newtonian fluid is higher (less) than the second-order fluid for net radial inflow (outflow) in the entire radial region.

If $s_1 = 1, s_2 = 0, \tau_1 = \tau_2 = 0$ and $s_1 = 1, s_2 = 0$, the results are in good agreement with those obtained by Sharma and Agarwal [5] and Sharma and Bhatia [10], respectively.

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On the effect of annealing on total normal emittance of oxidized steel

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INTRODUCTION

It is known that the radiative properties of metals are intimately related to the surface conditions because the penetration of electromagnetic waves is limited to a very thin layer. Hence, surface damage related to polishing processes and roughness and oxide layer thickness are very important parameters affecting both total and spectral emittance [1–6]. Further it has been shown [7] that when a metallic surface undergoes an oxidation process, previous polishing and its particular technique can considerably affect the oxide layer growth and therefore the surface's emittance. As a consequence, radiative properties of oxidized metals may largely depend upon the 'history' of the sample before oxidation.

The aim of this note is to investigate the influence of annealing on total normal emittance of oxidized steel samples.

EXPERIMENTAL RESULTS AND DISCUSSION

Total normal emittance tests have been carried out by the experimental set-up shown in Fig. 1 and previously described in ref. [8]. It allows a direct comparison of the total and spectral radiances issued by the surface under test and a blackbody cavity.

The two radiances are alternately focused onto the entrance slit of a grating monochromator by means of a system of mirrors L_1 , L_2 , L_3 . A high-vacuum Hilger-Schwartz thermopile collects the radiation emitted by the sample surface and by a blackbody cavity. A P.A.R. lock-in amplifier is used to measure the thermopile output.

The size of the blackbody opening (1×6 mm) and the cavity dimensions yield an estimated apparent emittance of 0.99. The temperature difference between the sample and the blackbody cavity is held within 1 K by means of an automatic temperature controlling system.

Before the test the experimental set-up was checked against spectral emittance standards (NBS). The difference between the measured and certified values fell within the standard deviations specified by NBS in the wavelength range $1\text{--}7\text{ }\mu\text{m}$.

Since the oxidation rate of a metallic surface is strongly affected by many parameters such as physico-chemical characteristics of the material, oxidation, temperature and surface status, it is very important, in order to study the influence of annealing, to control the surface finishing as much as possible.

In ref. [9] it was shown that the total normal emittance of this steel after oxidation in air was influenced by the initial surface roughness. An extremely low oxidation rate was observed on a polished mirror surface compared with that on surfaces with a greater initial roughness. On the other hand the same initial roughness does not involve a similar oxidation rate. In ref. [7] an increase of the average emittance by a factor of about 2.5 was observed when the sample surfaces were polished to the same final roughness by an industrial or a metallographic procedure. This effect, ascribed to surface damage can be removed by a suitable electrochemical polishing process.

In order to study the effect of annealing on the total normal emittance of oxidized steel, two series of ten samples each were obtained from a rolled-mill plate of AISI 316 L steel, and one of them was annealed at 1350 K in an argon atmosphere for 1 h.

All the specimens of the two series were then mirror polished

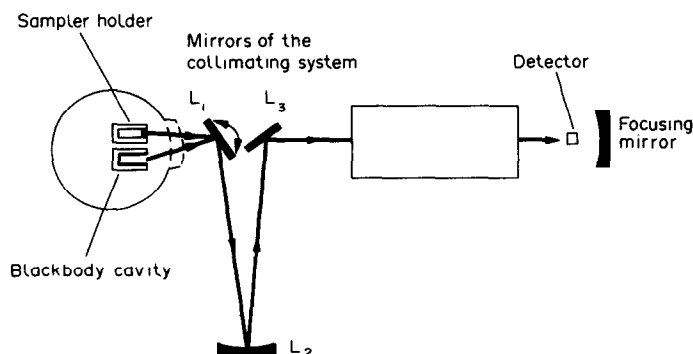


FIG. 1. Scheme of optical system.